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The Critical Flow Rate and Zero Entropy of Helium Superfluid

by

William Band
The Department of Physics
The State College of Washington
Pullman   Washington

The critical flow rate and zero entropy of helium superfluid

It is well known that the number of symmetrical eigenfunctions of an assembly of non-interacting Bose-Einstein systems with a distribution-in-energy specified by the set of numbers $\{n_j\}$ is

$$C = \prod_j \frac{(n_j + w_j - 1)!}{n_j!(w_j - 1)!} \tag{1}$$

where w_j is the degeneracy of the jth energy level. Assuming the eigenfunctions represent equally probable states, the assembly has entropy

$$S = . k \ln C \tag{2}$$

When the energy levels are non-degenerate

$$w_j = 1, \quad C = 1, \quad S = 0 \tag{3}$$

the entropy of every energy distribution is identically zero.

Ordinarily one maximizes $\ln C$ to find the most probable distribution-in-energy using either the Stirling approximation to the factorials, or the Darwin-Fowler method of steepest descents, and derives the familiar expression for the most probable distribution numbers:

$$\bar{n}_j = w_j / \{e^{(\epsilon_j - \mu)/kT} - 1\} \tag{4}$$

where T and μ are determined by the total energy and number, E, N. One then substitutes this into Eq.(1), uses the Stirling approximation and derives the entropy from Eq.(2), the result incidentally being

$$S = (E - N\mu)/T - k \sum_j w_j \ln\{1 - e^{(\mu - \epsilon_j)/kT}\} \tag{5}$$

If now one sets $w_j = 1$ in this result, it is by no means zero, and is therefore inconsistent with Eq.(3). In fact, while the distribution numbers of Eq.(4) are true for unit weights w_j, Eq.(5) is valid only when $w_j \gg 1$.

In principle, for a Bose-Einstein gas in any ordinary container, w_j is indeed unity, any degeneracy being accidental, and the conventional calculation of entropy is therefore strictly false. The conventional calculation is saved by the Heisenberg Uncertainty Principle (1). The life time τ of any system in any one state is so short that the uncertainty in energy is greater than the spacing $\triangle \in$ between the non-degenerate levels, and the spectrum is effectively continuous. One can then accept a large number of levels $w(\in)$ in a small energy range $d \in$ and assume $w(\in)d\in \gg 1$. This validates the usual value of entropy in Eq.(5).

If under certain circumstances τ becomes anomalously long, this validity breaks down: it is suggested that this is what happens in helium superfluid. If the superfluid forms in globules (2) of linear dimensions L cm. as a result of statistical fluctuations in population, the energy levels are spaced according to the Bose-Einstein model of F.London with intervals roughly given by

$$\triangle \in \quad \sim \quad h^2/8\pi m^*L^2 \qquad (6)$$

On the relaxation picture of thermal resistance in liquid helium II developed by Meyer and Band in 1948 (3), the life-time of an atom in the superfluid state is coupled with the flow rate of the superfluid by the relation

$$\tau u_s^2 \quad = \quad 0.05 \ cm^2/sec \quad at \quad 2^0 K \tag{7}$$

where u_s is the speed of the superfluid flow. If now

$$\Delta \epsilon \tau \ > \ h \tag{8}$$

the energy levels are too widely spaced for the uncertainty principle to blur them together and produce entropy. The entropy is therefore identically zero if

$$L u_s \ \leq \ 10^{-3} \ cm^2/sec \tag{9}$$

an equation obtained by substituting Eqs.(6) and (7) into (8) and using m^* equal approximately to $1.43 \ m_{He}$ for the effective mass.

Previous attempts to connect the critical flow rate with the uncertainty principle have been inspired by the observed fact that

$$L u_o \ \sim \ h/m_{He} \tag{10}$$

an equation that is numerically equivalent to Eq.(9). Since L and u_o are always perpendicular to each other, the uncertainty principle does not apply directly in the form of Eq.(10). Instead we are led by the present discussion to picture the filtering out of normal fluid at the entrance to a slit of width L as due to its ability to accept globules of superfluid at a speed not exceeding u_s given by Eq.(9). If the slit were larger, either normal fluid enters as such or the globules are too large to have zero entropy and therefore remain normal; if the speed were greater, then the life time τ is shortened by Eq.(7), and the levels blur together to restore normal entropy. At lower speeds and smaller

dimensions, the energy levels in the globules remain distinct and their entropy remains zero corresponding to the superfluid state.

A theoretical explanation of the relation (7), which is essential to the above reasoning, has been suggested before in terms of thermodynamics and three independent state variables for non equilibrium states.(4)

References

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2.   J.G.Dash,   Physical Review, 94, pp. 825, 1091, 1954.

3.   L. Meyer and W. Band,   Physical Review, 74, p.394, 1948.

4.   W. Band,   ONR report, June 10 1952, N7onr33005, NR 010-603.

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